Syntax of LTL

LTL is built up from a finite set of propositional variables *AP*, the logical operators ¬ and ∨, and the temporal modal operators X (some literature uses O or N) and U. Formally, the set of LTL formulas over *AP* is inductively defined as follows:

- if $p \in AP$ then *p* is an LTL formula;
- if ψ and φ are LTL formulas then ¬ψ, φ ∨ ψ, X ψ, and φ U ψ are LTL formulas.[7]

X is read as next and U is read as until. Other than these fundamental operators, there are additional logical and temporal operators defined in terms of the fundamental operators, in order to write LTL formulas succinctly. The additional logical operators are ∧, →, ↔, true, and false. Following are the additional temporal operators.

- G for always (globally)
- F for finally
- R for release
- W for weak until
- M for mighty release

Semantics of LTL

An LTL formula can be *satisfied* by an infinite sequence of truth valuations of variables in *AP*. These sequences can be viewed as a word on a path of a Kripke structure (an ω-word over alphabet 2^{AP}). Let $w = a_0, a_1, a_2, \ldots$ be such an ω-word. Let $w(i) = a_i$. Let $w^i = a_i, a_{i+1}, \ldots$, which is a suffix of *w*. Formally, the satisfaction relation ⊨ between a word and an LTL formula is defined as follows:

- $w \vDash p$ if $p \in w(0)$
- $w \vDash \neg\psi$ if $w \nvDash \psi$
- $w \vDash \varphi \vee \psi$ if $w \vDash \varphi$ or $w \vDash \psi$
- $w \vDash \mathbf{X}\,\psi$ if $w^1 \vDash \psi$ (in the ne**x**t time step ψ must be true)
- $w \vDash \varphi\,\mathbf{U}\,\psi$ if there exists $i \geq 0$ such that $w^i \vDash \psi$ and for all $0 \leq k < i$, $w^k \vDash \varphi$ (φ must remain true **u**ntil ψ becomes true)

We say an ω-word *w* satisfies an LTL formula ψ when $w \vDash \psi$. The ω-language $L(\psi)$ defined by ψ is $\{w \mid w \vDash \psi\}$, which is the set of ω-words that satisfy ψ. A formula ψ is *satisfiable* if there exist an ω-word *w* such that $w \vDash \psi$. A formula ψ is *valid* if for each ω-word *w* over alphabet 2^{AP}, we have $w \vDash \psi$.